

Correction to “Injective Stability for K_2 ”

(From old file. Not dated.)

The proof of Proposition 5.21, case 2, is stated incorrectly. It should read as follows:

Case2: $v_2 = 0$. We apply inv and discuss instead:

Case 2': $w_3 = 0$. We may now assume $1 + v_1q_1 \in R^*$.

Via inv and Lemma 3.34 we see that $\mathcal{R}(x_{42}(-q_2)x_{43}(-q_3)) \circ \mathcal{R}(x_1(w_2, 0, w_4)) = \mathcal{R}(x_1(w_2, 0, w_4 + q_2w_2)) \circ \mathcal{R}(x_{42}(-q_2)x_{43}(-q_3))$, so that we can get rid of q_2, q_3 . Say $q_2 = q_3 = 0$. Choose $\lambda \in R$ such that the top half of the first column of $\underline{\text{mat}}(\mathcal{L}(x_{23}(\lambda))\mathcal{L}(x_4(v))\mathcal{R}(x_1(w))\langle X, Y \rangle)$ is a unimodular row of length two. As $\underline{\text{mat}}(\mathcal{R}(x_1(w))\langle X, Y \rangle)$ has a trivial third row, it is easy to see that $\mathcal{L}(x_4(v_1, v_2 + \lambda v_3, 0))\mathcal{R}(x_1(w))\langle X, Y \rangle$ is defined. We may replace v by $(v_1, v_2 + \lambda v_3, v_3)$ provided that we replace $\langle X, Y \rangle$ by $\mathcal{L}(x_{23}(\lambda))\mathcal{R}(x_{23}(-\lambda))\langle X, Y \rangle$. In other words, we may assume that $\mathcal{L}(x_4(v_1, v_2, 0))\mathcal{R}(x_1(w))\langle X, Y \rangle$ is defined. Now $\mathcal{L}(x_4(v_1, v_2, 0))\langle X, Y \rangle$ has the form $\langle P', Q'Y \rangle$ with $P' \in \text{St}(\{1, 2, 4\} \times \{1, 2\})$, $Q' \in \text{St}(\{1, 2, 4\} \times \{2, 4\})$. From this one sees that the maps $\mathcal{L}(x_{34}(v_3)), \mathcal{R}(x_1(w))$ behave at $\mathcal{L}(x_4(v_1, v_2, 0))\langle X, Y \rangle$ as in the case $v_1 = v_2 = w_3 = 0$, which is Case 1 up to inv. So $\mathcal{R}(x_1(w))\mathcal{L}(x_4(v))\langle X, Y \rangle = \mathcal{R}(x_1(w))\mathcal{L}(x_{34}(v_3))\mathcal{L}(x_4(v_1, v_2, 0))\langle X, Y \rangle = \mathcal{L}(x_{34}(v_3))\mathcal{R}(x_1(w))\mathcal{L}(x_4(v_1, v_2, 0))\langle X, Y \rangle$. The result now follows from the squeezing principle with $i = 3$.